AP Calculus AB Name _____

Applications of the Derivative Test Review

1. A ball is projected straight up from a 256-foot building with an initial velocity of 96ft/sec. Find the following **without** the use of a calculator:

a. The position, velocity and acceleration functions
b. When the ball reaches its maximum height.
c. How high the ball goes.
d. When the ball hits the ground.
e. The impact velocity.
f. Total distance traveled by the ball in the first 5 seconds.
g. Displacement of the ball in the first 5 seconds.
h. The velocity when the ball is 144 feet in the air.

2. A spherical balloon is being filled with gas at a rate of 500 cubic cm / min. How fast is the radius of the balloon increasing when the radius is: A) 30 cm? B) 60 cm?

3. A conical tank is being drained at a rate of 5 ft^3 / sec. The tank has a base diameter of 20 feet and a depth of 14 feet. At what rate is the water level decreasing when the tank is only half full?

Find the following for #'s 4 and 5:

a. Coordinates of any relative extrema.
b. Coordinates of the absolute extrema.
c. Any intervals where $f(x)$ is increasing.
d. Any intervals where $f(x)$ is decreasing.
e. Any points of inflection.
f. Any intervals where $f(x)$ is concave up.
g. Any intervals where $f(x)$ is concave down.
h. Sketch a possible graph for $f(x)$.

4. Given: $f(x) = x^3 - 3x^2 - 24x + 2$ on the closed interval [-4, 8].

5. Given: $f(x) = \sqrt[3]{(x+2)^2}$ on the closed interval [-3, 6].

6. Find the number guaranteed by the mean value theorem for the function $f(x) = e^{\frac{x}{2}}$ on the closed interval [-1, 4].

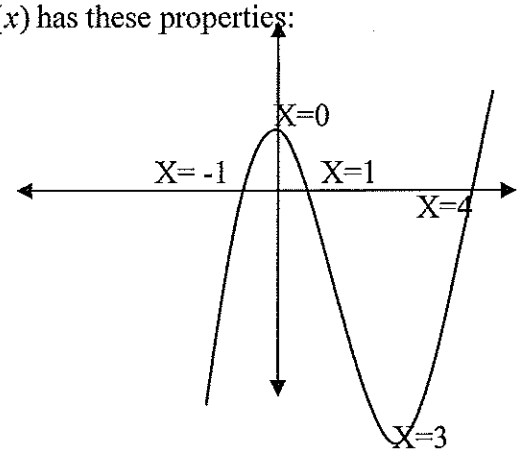
7. The following graph is the graph of $f'(x)$. The function $f(x)$ has these properties:

I. $f(x)$ has a domain of [-2,5].

II. $f(x)$ is everywhere differentiable on its domain.

III. $f(-2)=3$ and $f(5)=1$

IV. $f(x)$ has exactly four zeros in its domain.



(a) For what x coordinates if any does $f(x)$ have any relative extrema? Classify as a max or min and justify.

(b) For what x-coordinates if any does $f(x)$ have any points of inflection? Justify.

(c) Sketch a possible graph of $f(x)$ that fits the found information.

8. A plant storage area for a local nursery is to be constructed. The accountant for the nursery has stated that only $15,000 could be spent on the job. The storage area is to be rectangular in shape. One set of parallel sides must be constructed with a heavier grade of fencing in order to withstand the high traffic areas. The regular fence costs $4.00 per linear foot and the heavier grade fence costs $6.00 per linear foot. What are the dimensions of the storage area if the area is to be maximized?

9. A particle moves in a horizontal path such that its position is given by the function $s(t)=t^3-10t^2+27t-18$ where $S(t)$ is measured in meters and t is measured in seconds. Find the following:

a) What are the velocity and acceleration functions in terms of t?
b) When is the particle at rest?
c) What is the position(s) of the particle when at rest?
d) What is the position and velocity of the particle when the acceleration is -8 *m/s*?
e) Sketch a motion schematic for the particle. Make sure to label position and velocity at each critical time.
f) What was the total distance traveled in the first 5 seconds?
g) Find the displacement in the first 5 seconds.
h) What is the total distance traveled in the first 8 seconds?
i) Find the displacement in the first 8 seconds.
j) When is the particle moving to the right and left? Use interval notation for your answers.
k) What is the average velocity of the particle in the first 9 seconds?
l) What is the average velocity of the particle between the 3rd and 7th second?

10. The following is a graph of $f'(x)$. The function f has a domain of [-4, 3].

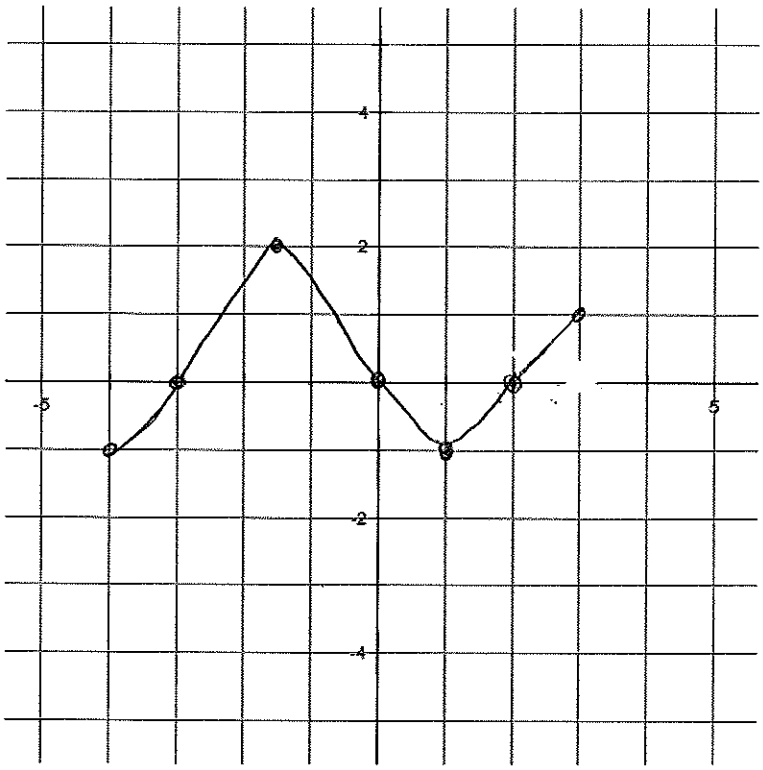
a) Over what x values is f increasing/decreasing? Justify your answer.

b) For what x values does the graph of f have any relative extrema? Classify as a max or
min and justify your answer.

c) Over what intervals is f concave up/down? Justify your answer.

d) For what x values does f have points of inflection? Justify your answer.

e) Sketch a possible graph of $f(x)$



11. A particle moves in a horizontal path such that its position at any time t in seconds is given by the function $s(t) = t^2 - 5t - \sin(2t) + 2$ where $s(t)$ is measured in feet. Find the following:

a) What are the velocity and acceleration functions?

b) When is the particle at rest?

c) What is the position of the particle when it is at rest?

d) What is the position of the particle when the velocity is 3 *ft/s*?

e) Over what intervals is the particle moving right or left?

f) Find the displacement in the first 7 seconds.

g) Find the total distance traveled in the first 7 seconds.

h) What is the average velocity of the particle in the first 8 seconds?

12. A 30 foot ladder leans against a house and begins to slide down the house at a rate of 2.8 ft/s. Answer the following:

A) How fast is the base of the ladder sliding away from the house when the top of the ladder is 4 ft from the ground?

B) How fast is the angle between the ground and the ladder changing when the base of the ladder is 15 ft from the house?

13. A farmer wants to enclose two adjacent pens for his horses. He needs to enclose a total area of 10,800 square feet. The outer fence will cost him $3 per foot and the inner fence will cost him $2 per foot. What dimensions should he use for each pen to minimize cost?

14. Car A drives toward a perpendicular intersection at 30 *mph*. Car B drives away from that same intersection at 54 *mph*.

A) At what rate is the distance between the cars changing when car A is 17 miles from the intersection and car B is 43 miles from the intersection?

B) At what rate is the angle between car A's path and the hypotenuse changing at the same time?

15. A cannon ball is fired vertically upward with a velocity of 276 *ft/s* from a cannon on a 73 foot cliff. Find the following:

A) Position, velocity and acceleration functions.
B) The maximum height of the cannon ball.
C) The velocity of the cannon ball when the ball is 100 feet off the ground.
D) The impact velocity of the ball.

16. A rectangular built in pool is to be designed with a surface area of 2400 square feet. There is to be a sidewalk around the pool that measures 2 feet on three sides and 3 feet on one of the shorter sides of the pool. What dimensions of the pool would minimize the area of the sidewalk (thereby minimizing the cost of the project).